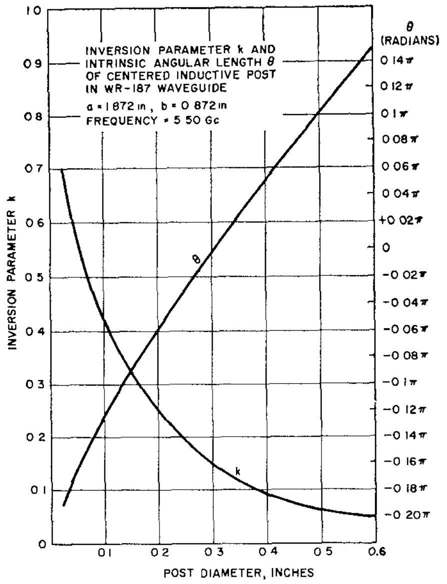

Fig. 4—Parameters of an inductive post in WR-187 waveguide.

where w and l are the amplitude and period shown in Fig. 3 Thus,

$$k = \tan \frac{\pi}{4}\left(1 - \frac{2w}{l}\right). \qquad (16)$$

Eq. (16) is a very practical expression, because plotting errors can be averaged out by using the ratio of the average amplitude $\bar{w}$ to the average period $\bar{l}$.

The intrinsic angular lengths, $\theta_1 = \beta_1 d_1$ and $\theta_2 = \beta_2 d_2$, are given by

$$\theta_1 = \beta_1 D_0 - \frac{\pi}{4}\left(1 - \frac{2w}{l}\right) \qquad (17)$$

$$\theta_2 = \beta_2 S_0 - \frac{\pi}{4}\left(1 - \frac{2w}{l}\right) \qquad (18)$$

where $\beta_1 D_0$ and $\beta_2 S_0$ are the coordinates of an "inside peak," such as point ②. The correct point to use is the one having the most nearly equal values of $\beta_1 D$ and $\beta_2 S$. If the network happens to have bilateral symmetry about plane T_0, $\beta_1 D_0$ and $\beta_2 S_0$ will be exactly equal; in general, they will differ by less than $\pi/2$.

An application for this measurement arises in the design of filters using thick inductive posts. The impedance behavior with frequency of a thick post does not follow that of a narrow iris, and the intrinsic angular length cannot be computed from the handbook value of the shunt susceptance. In Fig. 4 are shown values of θ and k derived from tangent-relation plots of data taken with centered inductive posts in WR-187 waveguide at 5.5 Gc. It is interesting that the sign of θ changes at a diameter of approximately $a/6$. For posts of larger diameter, $d_1 = d_2 = \theta/2\beta$ is positive, and directly-coupled resonator sections (of a cascade-resonator filter) must be slightly longer than $\lambda_{g0}/2$.

D. B. Churchill
Microwave Engineering Dept. D-40
Sperry Gyroscope Co.
Great Neck, N. Y.

Integral Quotient in Measurements of Ambipolar Diffused Plasma with TE_{011} Cavity

Despite some known exact solutions of plasma loaded TM_{010} cavity,[1,2] the TE_{011} mode should be used due to measurement-technical reaons in the case of large electron densities and considerable losses.[3,4] This communication is connected with the mathematical treatment of measurement results in the case of TE_{011} mode and ambipolar diffusion in discharge tube. The treatment is based on Slater's[5] expression for discharge admittance and the assumption of small perturbations, but not on any special electron theoretically derived plasma conductivity formula.

Owing to the ambipolar diffusion in the discharge tube, the distribution of electron density n along the radius r is

$$n = n_0 J_0(2.405 r/r_0) \qquad (1)$$

where n_0 is the electron density at the axis and r_0 the inside radius of the discharge tube. The plasma conductivity is proportional to the electron density and obeys the same distribution.

Starting from Slater's discharge admittance,[5] the expressions for the real part σ_{r0} and the imaginary part σ_{i0} of plasma conductivity at the axis can be derived and are

$$\begin{cases} \sigma_{r0} = (g_d \omega_0 \epsilon_0/\beta) Q \\ \sigma_{i0} = (-2\Delta\omega_0 \epsilon_0) Q \end{cases} \qquad (2)$$

where g_d is the discharge conductance, ω_0 the angular resonant frequency, $\Delta\omega_0$ the change of ω_0 due to the discharge plasma, ϵ_0 the dielectric constant of free space, β the factor depending on coupling between transmission line and cavity, and Q the quotient of two volume integrals.

In the case of ambipolar diffusion and TE_{011} cavity, after integration with respect to z and ϕ (in cylindrical coordinates)[6] one has

$$Q = Q(R/r_0) = Q_1(R)/Q_2(R/r_0, r_0) = \int_0^R r J_1^2(3.832 r/R) dr \Big/ \int_0^{r_0} r J_0(2.405 r/r_0) J_1^2(3.832 r/R) dr. \qquad (3)$$

R is the inner radius of the cavity.

The integration in the numerator of (3) can be performed simply and one has $Q_1(R) = \frac{1}{2} R^2 J_0^2(3.832)$. The values of $Q_2(R/r_0, r_0)$ at $r_0 = 1$ cm have been computed on an electronic digital computer. The results are presented in Table I. Q_2 can be found for other values of r_0 by noting that it is proportional to the square of r_0. The integral quotient Q is only the function of R/r_0 and is presented in Table I and in Fig. 1 for practical values of the argument R/r_0.

TABLE I
COMPUTED VALUES OF DENOMINATOR Q_2 AT $r_0 = 1$ CM AND INTEGRAL QUOTIENT Q AS FUNCTION OF R/r_0, THE RATIO OF CAVITY RADIUS TO DISCHARGE TUBE RADIUS

R/r_0	Q_2, cm²	Q
3	0.022358	32.650
4	0.013688	94.805
5	0.0091115	222.54
6	0.0064640	451.71
7	0.0048106	826.14
8	0.0037141	1397.6
9	0.0029514	2225.9
10	0.0024005	3378.8

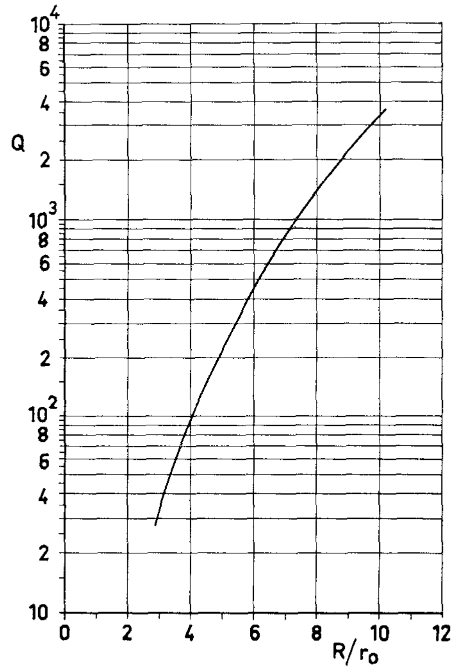

Fig. 1—The quotient Q of two volume integrals in the measurement of ambipolar diffused plasma with TE_{011} cavity. R is the inner radius of cavity, r_0 the inner radius of discharge tube.

Paavo Jääskeläinen
Electronics Div.
Finnish Cable Works Co.
Helsinki, Finland

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On "Status Report on International Millimeter Waveguide Flange Standards"[1]

In his communication, Anderson described the state of international standardization of millimeter waveguide flanges and concluded that in the absence of a suitable

Manuscript received May 14, 1964.

[1] T. N. Anderson, IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 427–429; September, 1963.